Muon capture on the deuteron and ^{3}He

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Abstract.
We plan to investigate the role of meson exchange currents in the description of the $\mu^- + d \rightarrow \nu_\mu + n + n$ and $\mu^- + {}^3\text{He} \rightarrow \nu_\mu + {}^3\text{H}$ reactions. They both are treated as the decay of the corresponding muonic atoms, with the muon initially on the lowest K shell. The muon binding energy in these atoms can be safely neglected and in the initial state we deal essentially with the deuteron (or ^{3}He) and muon at rest. These two reactions are interesting for several reasons. First of all, they offer a testing ground for the nuclear wave functions, which for any nucleon-nucleon (NN) and three-nucleon (3N) forces can be constructed for such light systems with great accuracy. In these reactions few-nucleon weak current operators are an important dynamical ingredient. In the current operators apart from the relatively well known single nucleon contributions, two-nucleon parts (generated by various meson exchanges) play an important role. Their details are not well known and several models should be considered. We present our formalism for dealing with these reactions and a simple method for partial wave decomposition of the two-nucleon operators. The crucial nuclear matrix elements of the corresponding weak current operators will be calculated in the momentum space and using partial wave decomposition. The effect of meson exchanges will be investigated in the energy spectrum of the emitted neutrinos (in the deuteron case) and in the total decay rates for the two reactions. We will employ various models of NN and 3N forces, such as the Bonn B or chiral NNLO potentials. Our results with the single nucleon currents look already very promising and we hope for the improvement in the description of the experimental data, when dominant two-nucleon current operators are included in our framework.

1 Introduction

We investigate the role of meson exchange currents in the description of muon capture reactions: $\mu^- + d \rightarrow \nu_\mu + n + n$ and $\mu^- + {}^3\text{He} \rightarrow \nu_\mu + {}^3\text{H}$. At the moment we can do it only implicitly, by comparing our predictions, based on the single nucleon current operator with the experimental data. In the future we will include explicit two-nucleon operators in our formalism. Muon capture on light nuclei is one of the best known examples of the weak interaction. Studying these processes enables us to understand the weak nuclear current operator, since ground and scattering states of few-nucleon systems are under control. Typically we study the decay of the muon atom and (in the deuteron case) deal with hyperfine structure of the atomic levels.

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Starting from the Fermi theory the Lagrangian density is

$$L(x) = -\frac{G}{\sqrt{2}} J_\lambda(x) J^{\lambda\dagger}(x) \tag{1}$$

and the full current is given by

$$J_\lambda(x) = l_\lambda(x) + j_\lambda(x). \tag{2}$$

The leptonic part (given here without the tau lepton) is well known:

$$l_\lambda(x) = \bar{\psi}_e(x)\gamma_\lambda(1-\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu\gamma_\lambda(1-\gamma_5)\psi_{\nu_\mu} \tag{3}$$

and the hadronic part $j_\lambda(x)$ is of main interest to us. We can apply this theory for the 2N and 3N system.

2 The single nucleon current operator

Matrix elements of the single nucleon current operator (see Fig. 1) are given in the momentum space in the following form for the density operator:

$$\langle \vec{p_1'}|j_0(1)|\vec{p_1}\rangle = g_1^V + g_1^A \frac{\sigma_1 \cdot (\vec{p_1} + \vec{p_1'})}{2m}\tau_- \tag{4}$$

and for the vector part:

$$\langle \vec{p_1'}|\vec{j}(1)|\vec{p_1}\rangle = g_1^V \frac{\vec{p_1} + \vec{p_1'}}{2m} - \frac{1}{2m}(g_1^V - 2m\, g_2^V)i\vec{\sigma_1} \times (\vec{p_1} - \vec{p_1'}) +$$
$$g_2^A(\vec{p_1} - \vec{p_1'})\frac{\vec{\sigma_1} \cdot (\vec{p_1} - \vec{p_1'})}{2m}\tau_- . \tag{5}$$

containing nucleon weak form factors, g_1^V, g_2^V, g_1^A, and g_2^A, which are functions of the four-momentum transfer squared, $(p_1' - p_1)^2$. On top of these strictly non-relativistic terms, one can also consider the so-called relativistic corrections.
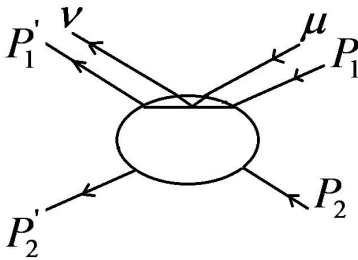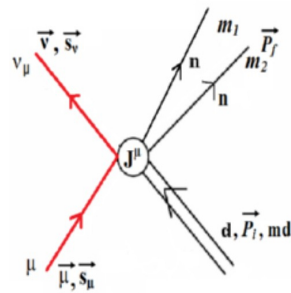
Figure 1. General single current diagram.　　**Figure 2.** Muon capture in deuteron.

3 Muon capture on the deuteron

The diagram corresponding to the $\mu^- + d \to \nu_\mu + n + n$ is shown in Fig. 2. The total decay rate reads:

$$
\Gamma_d = \frac{1}{2} G^2 \frac{1}{(2\pi)^2} \frac{\left(M'_d \alpha\right)^3}{\pi} \int_0^\pi d\theta_{p_\nu} \sin\theta_{p_\nu} \int_0^{2\pi} d\phi_{p_\nu} \int_0^{E_\nu^{max,nn}} dE_\nu E_\nu^2 \frac{1}{2} M_n p
$$

$$
\int_0^\pi d\theta_p \sin\theta_p \int_0^{2\pi} d\phi_p \frac{1}{6} \sum_{m_d, m_\mu} \sum_{m_1, m_2, m_\nu} \left| \ell_\lambda(m_\nu, m_\mu) N^\lambda(m_1, m_2, m_d) \right|^2 , \tag{6}
$$

where the factor $\frac{\left(M'_d \alpha\right)^3}{\pi}$ stems from the K-shell atomic wave function, $M'_d = \frac{M_d M_\mu}{M_d + M_\mu}$ and $\alpha \approx \frac{1}{137}$ is the fine structure constant.
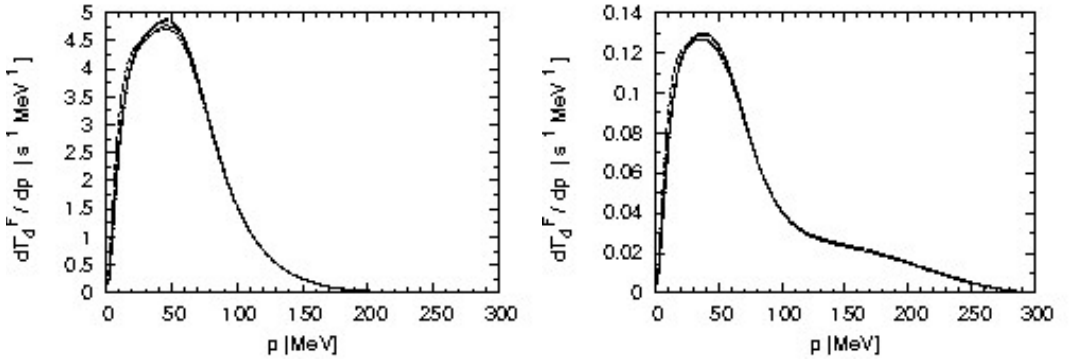

Figure 3. Differential capture rate $d\Gamma_d^F/dp$ of the $\mu^- + d \to \nu_\mu + n + n$ process calculated using standard PWD with various nucleon-nucleon potentials: the AV18 potential [8] (solid curves), the Bonn B potential [6] (dashed curves) and the set of chiral NNLO potentials from Ref. [5] (bands) for $F = \frac{1}{2}$ (left panel) and $F = \frac{3}{2}$ (right panel) as a function of the relative neutron-neutron momentum p. Note that the bands are very narrow and thus appear practically as a curve. All the partial wave states with $j \leq 4$ have been included in the calculations with the single nucleon current operator containing the relativistic corrections. Note that the average "nucleon mass" is used in the kinematics and in solving the Lippmann-Schwinger equations.

4 Muon capture on ^{3}He

In the 3N system we restrict ourselves to the $\mu^- + {}^3\text{He} \to \nu_\mu + {}^3\text{H}$ reaction channel. In this case the final 3N state is bound and the total decay rate can be written as

$$
\Gamma_{^3\text{H}} = \frac{1}{2} G^2 \frac{1}{(2\pi)^2} \mathcal{R} \frac{\left(2M'_{^3\text{He}} \alpha\right)^3}{\pi} \rho
$$

$$
4\pi \frac{1}{2} \sum_{m_{^3\text{He}}} \sum_{m_{^3\text{H}}} \left(\left| N^0(m_{^3\text{H}}, m_{^3\text{He}}) \right|^2 + \left| N_z(m_{^3\text{H}}, m_{^3\text{He}}) \right|^2 + \right.
$$

$$
\left. 2 \left| N_{-1}(m_{^3\text{H}}, m_{^3\text{He}}) \right|^2 + 2\text{Re}\left(N^0(m_{^3\text{H}}, m_{^3\text{He}}) (N_z(m_{^3\text{H}}, m_{^3\text{He}}))^* \right) \right), \tag{7}
$$

Table 1. Doublet ($F = 1/2$) and quadruplet ($F = 3/2$) capture rates for the $\mu^- + d \to \nu_\mu + n + n$ reaction calculated with various nucleon-nucleon potentials and the single nucleon current operator without and with the relativistic corrections (RC). Plane wave results (PW) and results obtained with the rescattering term in the nuclear matrix element (full) are shown.

| | Capture rate Γ_d^F in s^{-1} | | | |
| | $F = 1/2$ | | $F = 3/2$ | |
nucleon-nucleon force and dynamics	PW	full	PW	full
Bonn B, without RC	369	403	10.0	11.7
Bonn B, with RC	363	396	10.4	12.2
AV18, with RC	361	392	10.2	12.0
experimental results:				
I.-T. Wang *et al.* [1]	365 $\pm$ 96			
A. Bertin *et al.* [2]	445 $\pm$ 60			
G. Bardin *et al.* [3]	470 $\pm$ 29			
M. Cargnelli *et al.* [4]	409 $\pm$ 40			

where the factor $\dfrac{\left(2M'_{^3\mathrm{He}}\alpha\right)^3}{\pi}$, like in the deuteron case, comes from the K-shell atomic wave function and $M'_{^3\mathrm{He}} = \dfrac{M_{^3\mathrm{He}}M_\mu}{M_{^3\mathrm{He}}+M_\mu}$. The additional factor $\mathcal{R}$ accounts for the finite volume of the ^{3}He charge and we assume that $\mathcal{R} = 0.98$ [7].
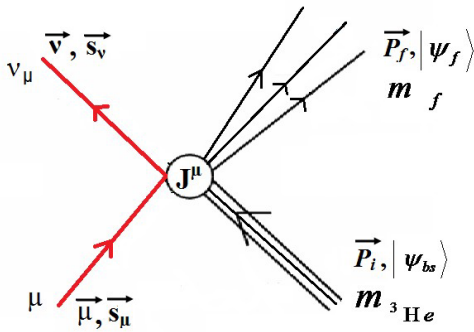

Figure 4. Muon capture in ^{3}He.

5 Results and conclusions

Our calculations are done for the single nucleon current operator and the calculations using 2N operators are in progress. We present results obtained within two approaches: without and with relativistic corrections. The capture rates results for the $\mu^- + d \to \nu_\mu + n + n$ reaction are calculated for both doublet ($F = 1/2$) and quadruplet ($F = 3/2$) capture rates (see Table 1). Also the results for the capture rates in the $\mu^- + {}^3\mathrm{He} \to \nu_\mu + {}^3\mathrm{H}$ case are shown in Table 2. In our calculations new methods have been developed based on an intensive use of Mathematica ®. We would like to inform the reader that the theoretical error caused by neglecting all contributions beyond the single nucleon term.

Table 2. Total capture rate Γ for the $\mu^- + {}^3\text{He} \to \nu_\mu + {}^3\text{H}$ reaction calculated with the single nucleon current operator and various nucleon-nucleon potentials. In the last two lines the rates are obtained employing the AV18 [8] nucleon-nucleon and the Urbana IX 3N potential [9], and adding, in the last line, some selected 2Ns current operators to the single nucleon current (see text for more explanations).

Three-nucleon Hamiltonian	Capture rate Γ in s^{-1}
Bonn B	1360
chiral NNLO version 1	1379
chiral NNLO version 2	1312
chiral NNLO version 3	1350
chiral NNLO version 4	1394
chiral NNLO version 5	1332
AV18	1353
AV18 + Urbana IX	1324
AV18 + Urbana IX with MEC [10]	1386
The experimental value [11]	1496

Acknowledgements

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